

**MATHEMATICS  
STANDARD LEVEL  
PAPER 2**

Friday 8 May 2009 (morning)

1 hour 30 minutes

Candidate session number

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**INSTRUCTIONS TO CANDIDATES**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

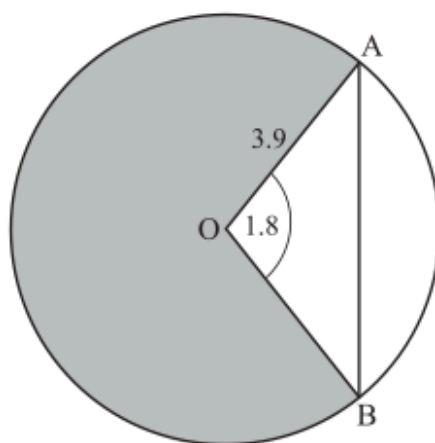
In an arithmetic series, the first term is  $-7$  and the sum of the first 20 terms is 620.

(a) Find the common difference. [3 marks]

(b) Find the value of the 78<sup>th</sup> term. [2 marks]

2. [Maximum mark: 7]

The circle shown has centre O and radius 3.9 cm.



*diagram not to scale*

Points A and B lie on the circle and angle AOB is 1.8 radians.

(a) Find AB. [3 marks]

(b) Find the area of the shaded region. [4 marks]

3. [Maximum mark: 6]

Let  $f(x) = \frac{3x}{2} + 1$ ,  $g(x) = 4 \cos\left(\frac{x}{3}\right) - 1$ . Let  $h(x) = (g \circ f)(x)$ .

- (a) Find an expression for  $h(x)$ . [3 marks]
- (b) Write down the period of  $h$ . [1 mark]
- (c) Write down the range of  $h$ . [2 marks]

4. [Maximum mark: 6]

A random variable  $X$  is distributed normally with mean 450 and standard deviation 20.

(a) Find  $P(X \leq 475)$ . [2 marks]

(b) Given that  $P(X > a) = 0.27$ , find  $a$ . [4 marks]

5. [Maximum mark: 6]

Two lines with equations  $\mathbf{r}_1 = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + s \begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix}$  and  $\mathbf{r}_2 = \begin{pmatrix} 9 \\ 2 \\ 2 \end{pmatrix} + t \begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix}$  intersect at the point P. Find the coordinates of P.

6. [Maximum mark: 7]

In a geometric series,  $u_1 = \frac{1}{81}$  and  $u_4 = \frac{1}{3}$ .

(a) Find the value of  $r$ . [3 marks]

(b) Find the smallest value of  $n$  for which  $S_n > 40$ . [4 marks]

7. *[Maximum mark: 8]*

In any given season, a soccer team plays 65 % of their games at home.  
When the team plays at home, they win 83 % of their games.  
When they play away from home, they win 26 % of their games.

The team plays one game.

- (a) Find the probability that the team wins the game. *[4 marks]*
- (b) If the team does not win the game, find the probability that the game was played at home. *[4 marks]*



Do **NOT** write on this page.

### Section B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

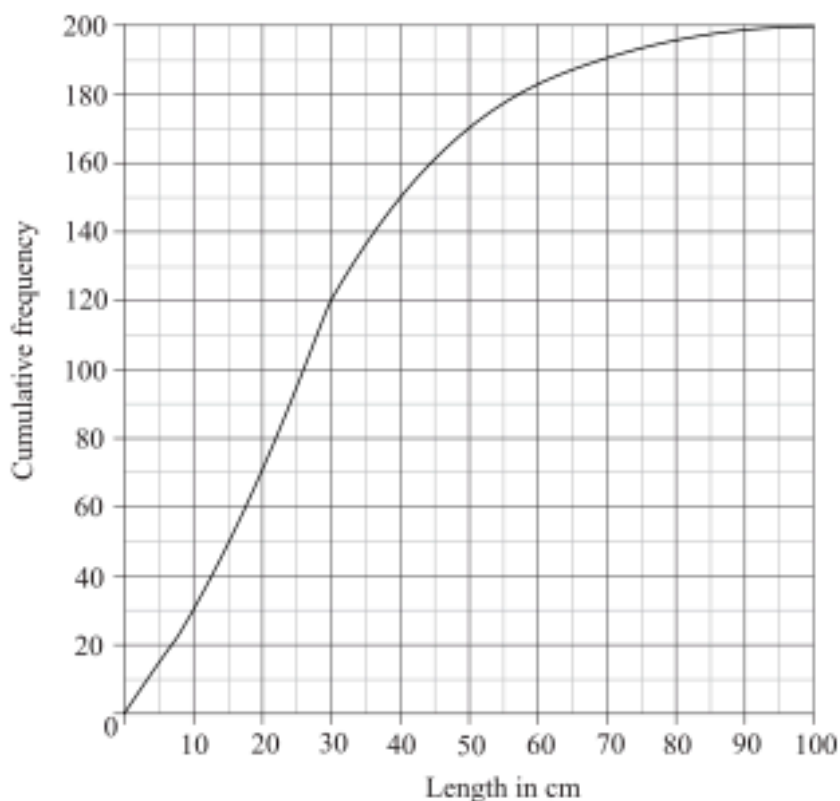
8. [Maximum mark: 15]

A fisherman catches 200 fish to sell. He measures the lengths,  $l$  cm of these results are shown in the frequency table below.

Length $l$ cm	$0 \leq l < 10$	$10 \leq l < 20$	$20 \leq l < 30$	$30 \leq l < 40$	$40 \leq l < 60$	$60 \leq l < 75$	$75 \leq l < 100$
Frequency	30	40	50	30	33	11	6

(a) Calculate an estimate for the standard deviation of the lengths of the fish. [3 marks]

(b) A cumulative frequency diagram is given below for the lengths of the fish.



Use the graph to answer the following.

- (i) Estimate the interquartile range.
- (ii) Given that 40 % of the fish have a length more than  $k$  cm, find the value of  $k$ .

[6 marks]

In order to sell the fish, the fisherman classifies them as small, medium or large.

Small fish have a length less than 20 cm.

Medium fish have a length greater than or equal to 20 cm but less than 60 cm.

Large fish have a length greater than or equal to 60 cm.

- (c) Write down the probability that a fish is small. *[2 marks]*

The cost of a small fish is \$4, a medium fish \$10, and a large fish \$12.

- (d) Copy and complete the following table, which gives a probability distribution for the cost \$ $X$ . *[2 marks]*

cost \$ $X$	4	10	12
$P(X = x)$		0.565	

- (e) Find  $E(X)$ . *[2 marks]*

9. [Maximum mark: 15]

Let  $f(x) = ax^2 + bx + c$  where  $a$ ,  $b$  and  $c$  are rational numbers.

(a) The point  $P(-4, 3)$  lies on the curve of  $f$ . Show that  $16a - 4b + c = 3$ . [2 marks]

(b) The points  $Q(6, 3)$  and  $R(-2, -1)$  also lie on the curve of  $f$ . Write down two other linear equations in  $a$ ,  $b$  and  $c$ . [2 marks]

(c) These three equations may be written as a matrix equation in the form  $\mathbf{AX} = \mathbf{B}$ ,

where  $\mathbf{X} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ .

(i) Write down the matrices  $\mathbf{A}$  and  $\mathbf{B}$ .

(ii) Write down  $\mathbf{A}^{-1}$ .

(iii) **Hence** or otherwise, find  $f(x)$ . [8 marks]

(d) Write  $f(x)$  in the form  $f(x) = a(x-h)^2 + k$ , where  $a$ ,  $h$  and  $k$  are rational numbers. [3 marks]

10. [Maximum mark: 15]

Let  $f(x) = x^3 - 4x + 1$ .

(a) Expand  $(x+h)^3$ .

[2 marks]

(b) Use the formula  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  to show that the derivative of  $f(x)$  is  $3x^2 - 4$ .

[4 marks]

(c) The tangent to the curve of  $f$  at the point  $P(1, -2)$  is parallel to the tangent at a point  $Q$ . Find the coordinates of  $Q$ .

[4 marks]

Adobe

(d) The graph of  $f$  is decreasing for  $p < x < q$ . Find the value of  $p$  and of  $q$ .

[3 marks]

(e) Write down the range of values for the gradient of  $f$ .

[2 marks]